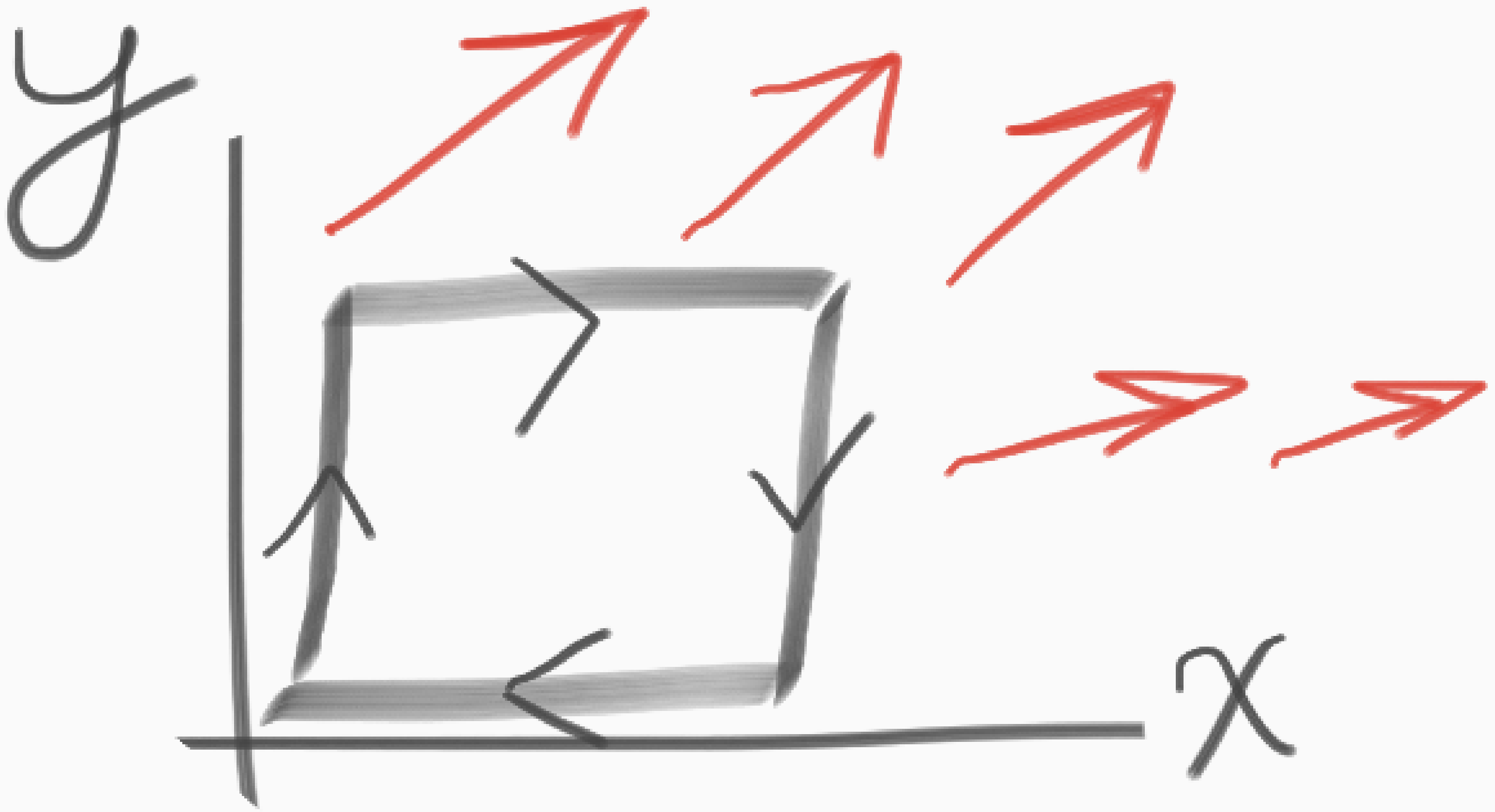
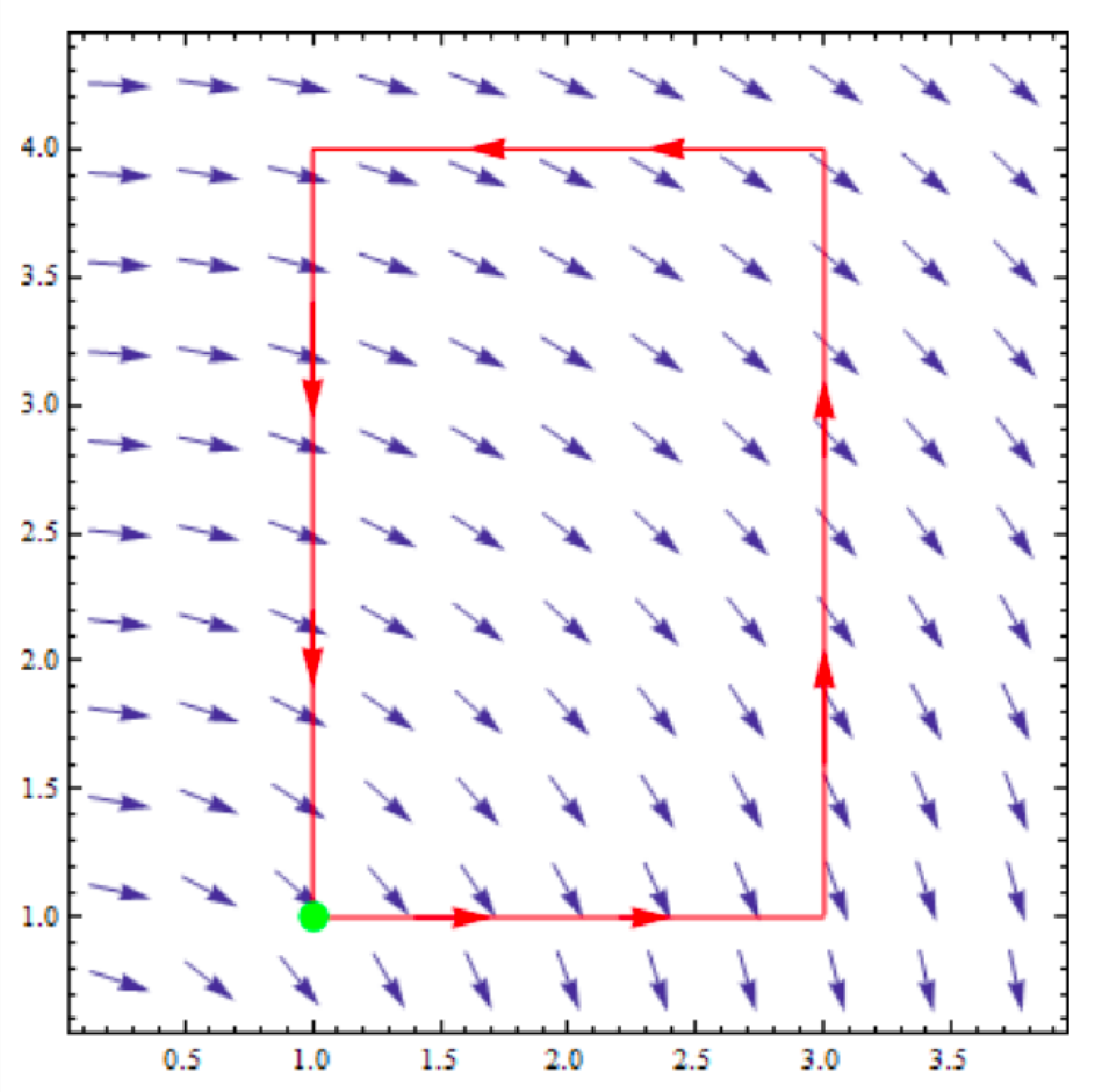
$$F(x,y) = \begin{bmatrix} P(x,y) \\ Q(x,y) \end{bmatrix}$$

$$\oint_C \vec{F} \cdot d\vec{r} = \iint_D (\nabla \times F) \cdot k \, dA$$

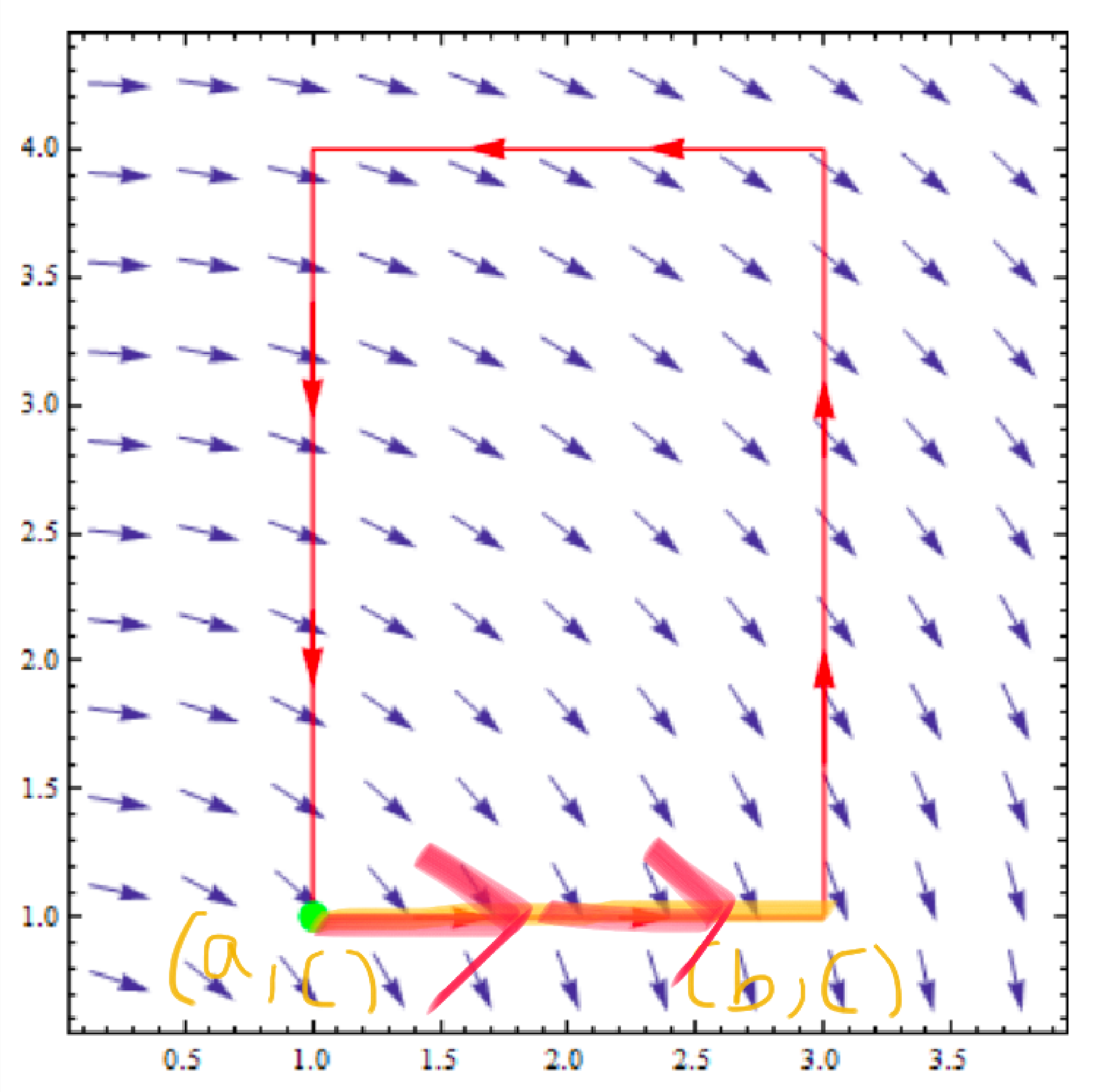
Line integral is a measure of how well a vector field travels with curve



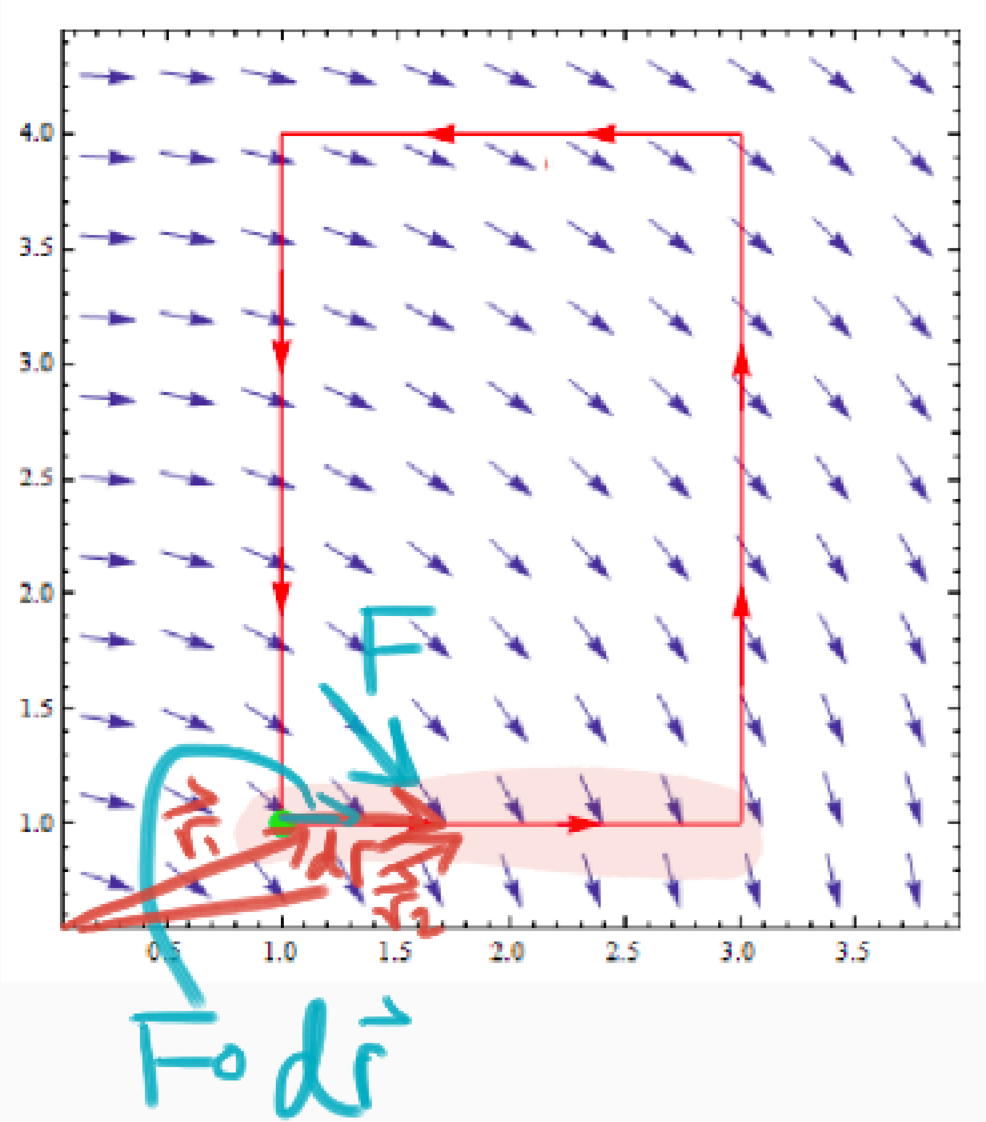
$4x^2$



*Work done on a closed path on a conservative vector field (i.e., Gravitational Field) is 0



$$D = \{(x,y) : a \le x \le b,\ c \le y \le d$$



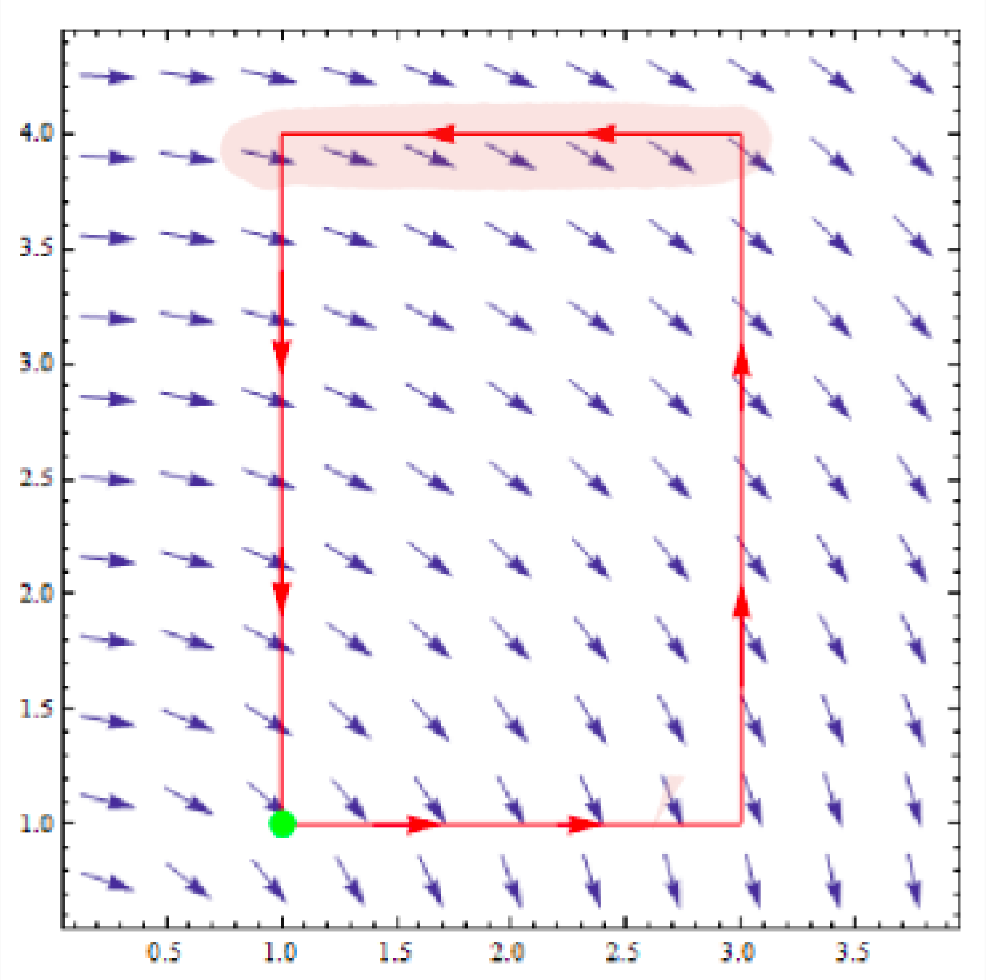
Goal: Find work done on green marble on curve C by vector field F

$$F(x,y)=\begin{bmatrix} P(x,y) \\ Q(x,y) \end{bmatrix}$$

$$\oint_C \vec{F} \circ d\vec{r} = \oint_C \vec{F} \circ \hat{r}\, dr$$

$$\hat{r} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \circ \begin{bmatrix} P(x,y) \\ Q(x,y) \end{bmatrix} = P(x,y)$$

$$\oint_C P(x,y)\, dr = \boxed{\oint_C P(x,c)\, dr}$$



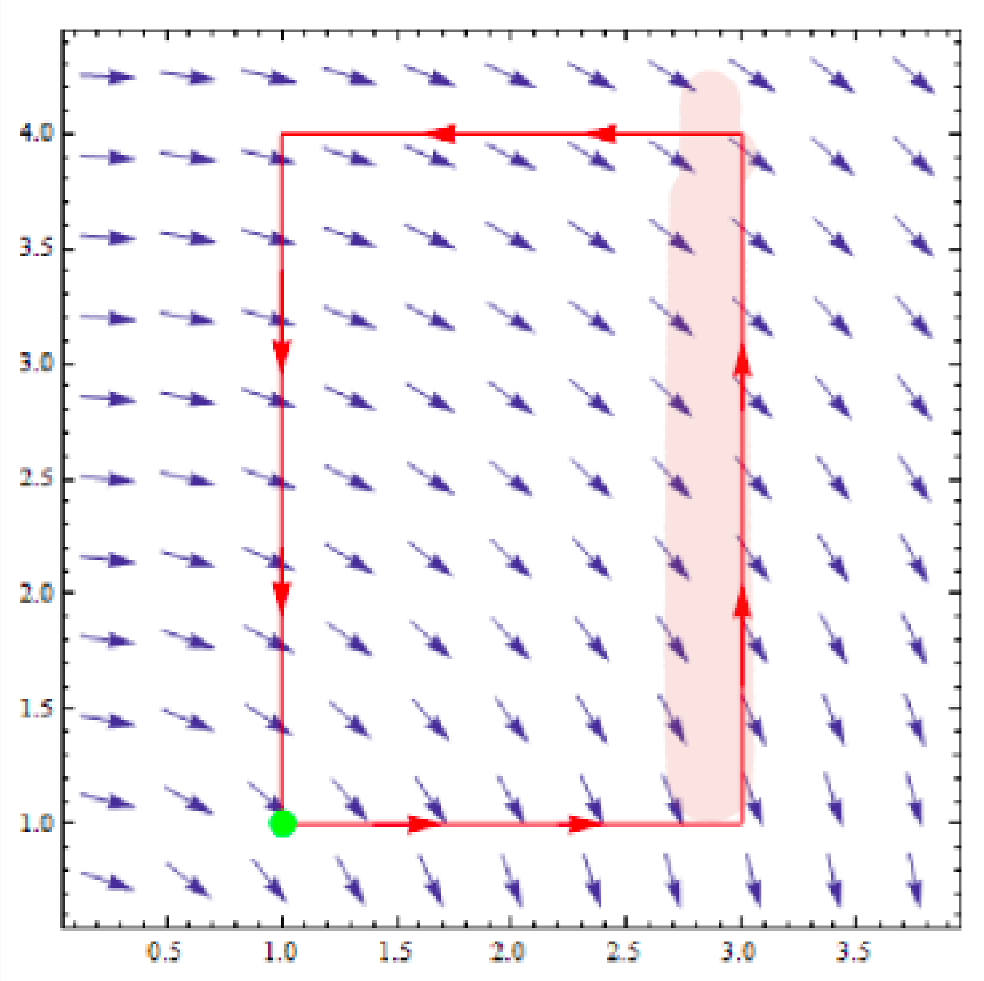
Goal: Find work done on green marble on curve C by vector field F

$$F(x,y) = \begin{bmatrix} P(x,y) \\ Q(x,y) \end{bmatrix}$$

$$\oint_C \vec{F} \circ dr = \oint_C \vec{F} \circ \hat{r}\, dr$$

$$\vec{F} \circ \hat{r} = \begin{bmatrix} P \\ Q \end{bmatrix} \circ \begin{bmatrix} -1 \\ 0 \end{bmatrix} = -P(x,y)$$

$$\oint_C -P(x,y)\, dr = \boxed{\oint_C -P(x,4)\, dr}$$



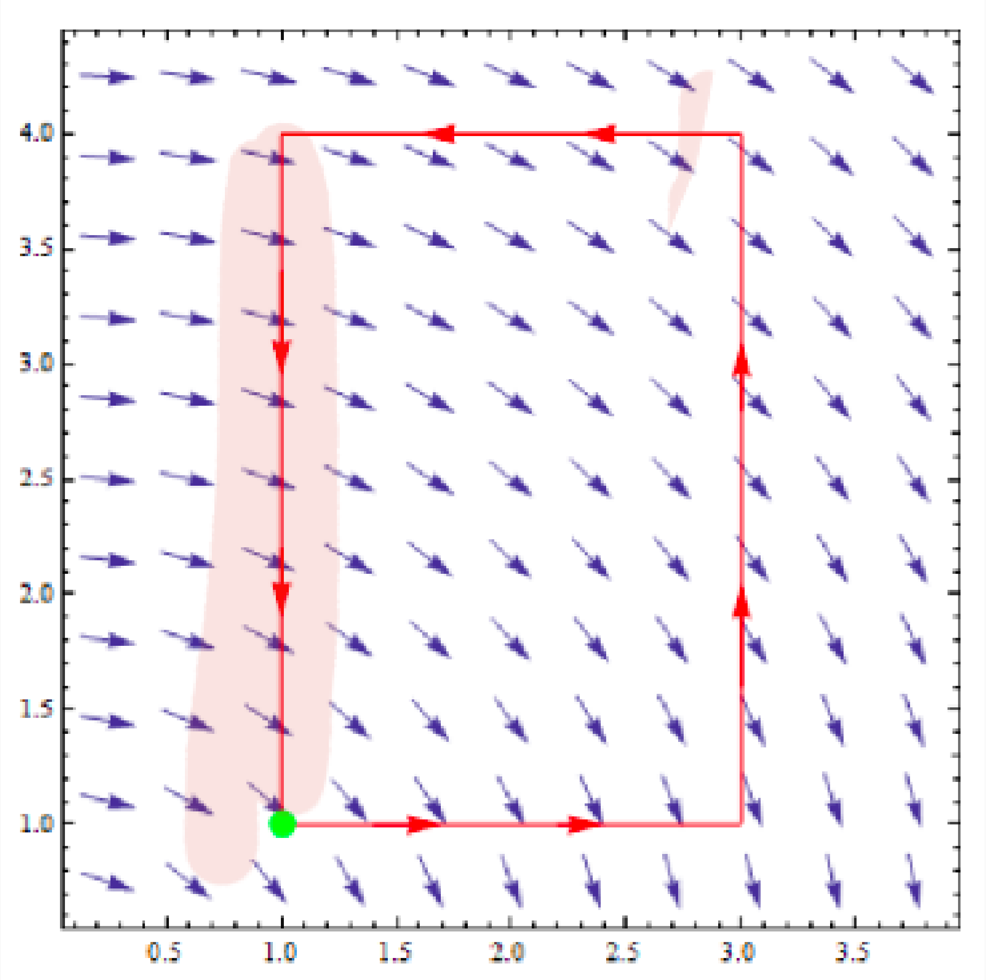
Goal: Find work done on green marble on curve C by vector field F

$$F(x,y)=\begin{bmatrix}P(x,y)\\Q(x,y)\end{bmatrix}$$

$$\oint_C \vec{F}\circ dr=\oint_C \vec{F}\circ\hat{r}\,dr$$

$$r\circ F=\begin{bmatrix}0\\1\end{bmatrix}\circ\begin{bmatrix}P\\Q\end{bmatrix}=Q(x,y)$$

$$\boxed{\oint_C Q(a,y)\,dr}$$

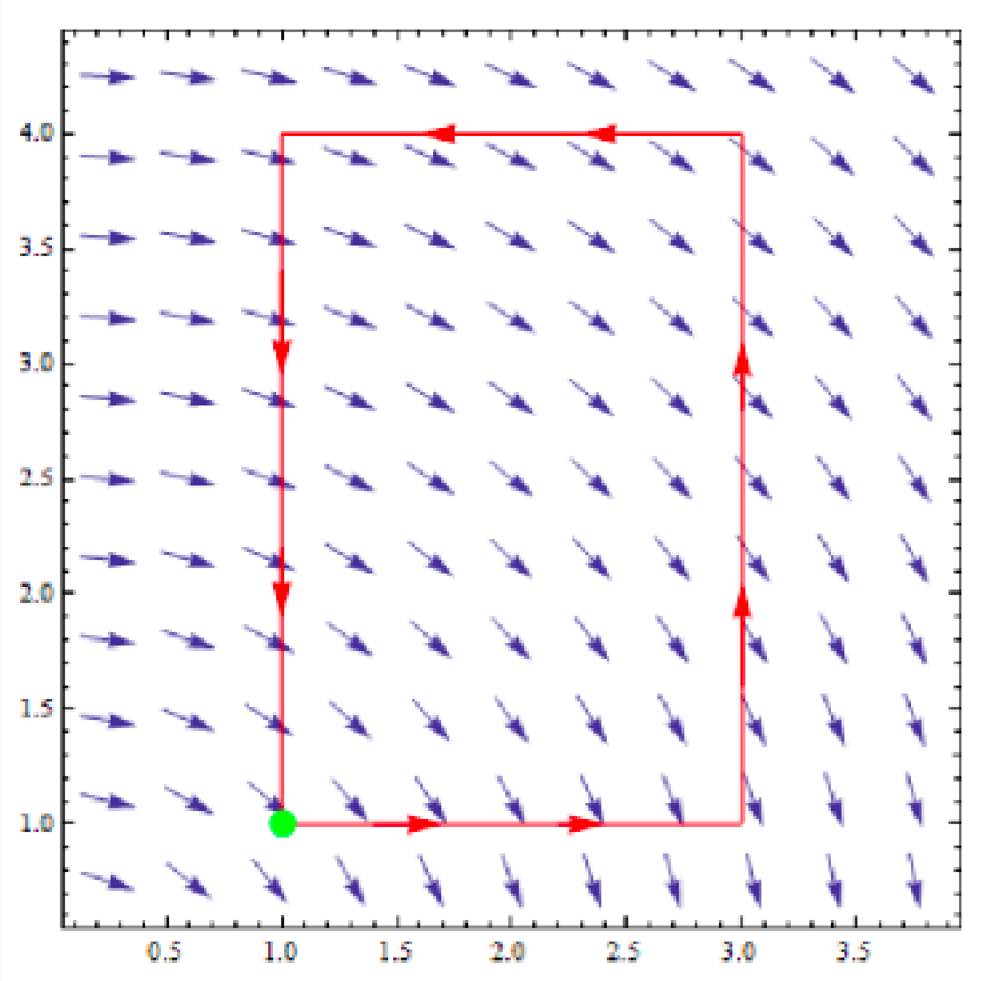


Goal: Find work done on green marble on curve C by vector field F

$$F(x,y) = \begin{bmatrix} P(x,y) \\ Q(x,y) \end{bmatrix}$$

$$\oint_C \vec{F} \circ dr = \oint_C \vec{F} \circ \hat{r}\, dr$$

$$\boxed{\oint_C -Q(x,y)\, dr}$$



Goal: Find work done on green marble on curve C by vector field F

$$F(x,y) = \begin{bmatrix} P(x,y) \\ Q(x,y) \end{bmatrix}$$

$$\oint_C \vec{F} \circ dr = \oint_C \vec{F} \circ \hat{r}\, dr$$

$$\int P(x,c) - P(x,d) + Q(a,y)$$

$$dr \qquad - Q(x,y)$$

Example 1

Using Green's theorem, evaluate the line integral $\oint\limits_C xydx + (x+y)\,dy$, where C is the curve bounding the unit disk R.

Example 2

Using Green's formula, evaluate the line integral $\oint\limits_C (x - y)\,dx + (x + y)\,dy$, where C is the circle $x^2 + y^2 = a^2$.

Example 3

Using Green's theorem, calculate the integral $\oint\limits_C x^2y\,dx - xy^2\,dy$. The curve C is the circle $x^2 + y^2 = a^2$ (Figure 1), traversed in the counterclockwise direction.

Example 4

Using Green's formula, evaluate the integral $\oint\limits_C (x+y)\,dx - (x-y)\,dy$, where the curve C is the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (Figure 2).

Example 5

Using Green's formula, calculate the line integral $\oint\limits_C y^2dx + (x+y)^2dy$, where the contour C is the triangle ABD with vertices $A\left(a,0\right)$, $B\left(a,a\right)$, $D\left(0,a\right)$ (Figure 3).